

Application of the Bootstrap Statistical Method in Deriving Vibroacoustic Specifications

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Introduction

Background

Numerous types of spaceflight hardware are exposed to random vibroacoustic excitation during the three distinct launch events of liftoff, transonic flight and maximum dynamic pressure. Random acoustic levels can exceed 160 dB (referenced to 20 μ Pascals) in overall sound pressure level on large launch vehicles. The nonstationary vibroacoustic launch environments are usually analyzed separately, to determine the maximum dynamic loading, for each of the three launch events. Acoustic loads occurring at liftoff are generated by the turbulent mixing of the rocket engine exhaust gases with the atmosphere. Acoustic data measured during several liftoff events for the Titan IV launch vehicle will serve as the data source for this paper's main analytical topic.

During launch, part of the acoustic energy external to the vehicle is transmitted through the vehicle's outer protection layer. The ensuing internal acoustics excites the payload, resulting in high structural vibration responses of the payload. Payloads with large areas and low masses are particularly susceptible to this acoustic excitation. Delicate optical or electronic flight hardware may also be damaged by this acoustic excitation.

In order to ensure mission success, it is necessary for the National Aeronautics and Space Administration (NASA) to determine this internal acoustic environment for the launch vehicles utilized. Knowledge of these environments allows the spaceflight hardware to be properly designed for the structural loading that is experienced during launch. Normally, prior to launch the spaceflight hardware will undergo dynamic testing on the ground to specified vibroacoustic test levels, thereby providing added confidence for mission success.

Nonstationary Random Vibroacoustic Data Analysis

Both external (to the launch vehicle) and internal (inside an expendable launch vehicle's payload fairing (PLF) or inside the Shuttle's cargo bay) microphone measurements of the acoustic pressures may be obtained during launch. Measured pressure time histories, $x(t)$, are time-varying data from a nonstationary random process. Because each measured signal comes from a random (non-deterministic) source it cannot be described by a deterministic expression. If the source was stationary random, it could be described by statistics such as its mean (μ_x), and rms (root-mean-square) value (Ψ_x), which would be temporally invariant. However, these statistical values are time varying for nonstationary random sources. It is often convenient to estimate these time-varying averages over short, contiguous segments of the measured signal to obtain running averages.

For example, the one-third octave band (OTOB) sound pressure level (SPL) of a stationary pressure time history, $x(t)$ (where x equals pressure), is given in dB by:

$$L_x(f_i) = 10 \log_{10} \left(\frac{\Psi_x(f_i)}{\Psi_{ref}} \right)^2 \quad (1)$$

where $\Psi_x(f_i)$ is the rms value of the pressure signal $x(t)$ filtered through a one-third octave bandpass filter centered on frequency f_i and Ψ_{ref} is reference rms pressure (20 μ Pa, here).

For a nonstationary pressure source a time-varying SPL spectrum may be estimated from a running average of time-varying rms value, obtained by replacing $\Psi_x(f_i)$ in equation (1) by

$$\hat{\Psi}_x(f_i, t) = \left[\frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x^2(f_i, t) dt \right]^{\frac{1}{2}} \quad (2)$$

The hat (^) denotes an estimate, $x^2(f_i, t)$ is the instantaneous squared value of the pressure signal passed through the i th one-third octave bandpass filter, and T is the averaging time of analysis. For launch vehicle vibroacoustic data, $T = 1.0$ sec with 50 percent overlapping has been found to yield reasonable estimates that balance random and bias errors (ref. 1).

A composite spectrum, referred to as the “maximax” spectrum, is obtained by selecting at each OTOB frequency the largest value from all the time-varying SPL spectra, regardless of time slice of origin. This maximax SPL spectrum does not represent the instantaneous SPL at any specific time but instead has been found to provide a conservative measure of the dynamic environment with respect to the damage potential of this signal to spaceflight structures and equipment. A degree of conservatism (perhaps substantial) is incorporated into the analysis with the use of maximax spectrum. References 1 and 2 are excellent sources of information regarding the acquisition and analysis of dynamic data.

Vibroacoustic Test Specification

To properly test qualify spaceflight hardware to its launch acoustic environment, test levels are set based upon the appropriate maximax SPL spectra available. Zones are defined in a launch vehicle within which it is expected that acoustic environments will be reasonably similar.

Different launch vehicles of a particular type display flight-to-flight variations. Some of this variability is due to inherent differences between the flights such as different launch pads, payload configurations, and weights. But some of this variability is due to the randomness of the launch event itself, such as a hot engine burn or a three-sigma maximum dynamic pressure event. Due to limited available flight data, it is typical to include in the database as much flight data as is reasonably possible, to capture true variability.

Given a set of data, levels are typically derived that represent the maximum expected environment (MEE). This is a level that would typically not be exceeded, and should account for both the expected spatial variation within a particular zone as well as the known flight-to-flight variation. A second higher level denoted as the Extreme Expected Environment (EEE) is a level that should not be exceeded except for the most extreme circumstances. The EEE level is meant to cover known and unknown failure modes due to peak loading.

This paper illustrates two methods to calculate the MEE and EEE test levels. They are: (1) the Normal Tolerance Limit method (NTL) and (2) the Bootstrap method. The NTL has traditionally been utilized by NASA to calculate its MEE and EEE levels that are used for acceptance and qualification testing, respectively. The Bootstrap method is a statistical subsampling method that has wide use in many disciplines but has not been used for this application. Both methods will be applied to a set of Titan IV liftoff launch vehicle acoustic data and their respective results will be compared in this paper.

NASA's Traditional Method of Setting Vibroacoustic Test Levels

Normal Tolerance Limit Method

Numerous methods could be applied to a set of measured data to compute vibroacoustic test levels. NASA has traditionally used what is called the (NTL) method to compute vibroacoustics test levels (ref. 3).

Normal tolerance limits should be applied only to normally distributed random variables. If this assumption is appropriate, then the normal tolerance limit (NTL_x) for the set of x variables, x_i ; $i = 1, 2, \dots, n$, is given by

$$\text{NTL}_x(n, \beta, \gamma) = \bar{X} + (K_{n, \beta, \gamma} * s_x) \quad (3)$$

where

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n x_i \quad s_x = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^n (x_i - \bar{X})^2} \quad (4)$$

are the sample mean and standard deviation of x , respectively, $K_{n, \beta, \gamma}$ is the normal tolerance factor, β is the minimum portion (probability) of all values that will be less than NTL_x(n, β, γ), and γ is the confidence coefficient associated with NTL_x(n, β, γ).

The K normal tolerance factors may be easily found in reference 4. A subset of these K factors is provided in table 1. The magnitude of the K factor is affected by both the probability desired and the confidence desired. This uncertainty in the confidence results from using a sample mean and sample standard deviation in lieu of the population's true mean and standard deviation values.

TABLE 1.—NORMAL TOLERANCE K FACTORS

n	$\gamma = 0.50$			$\gamma = 0.75$			$\gamma = 0.90$		
	$\beta = 0.90$	$\beta = 0.95$	$\beta = 0.99$	$\beta = 0.90$	$\beta = 0.95$	$\beta = 0.99$	$\beta = 0.90$	$\beta = 0.95$	$\beta = 0.99$
10	1.32	1.70	2.41	1.67	2.10	2.93	2.06	2.57	3.53
17	1.31	1.68	2.37	1.55	1.96	2.74	1.82	2.27	3.14
50	1.29	1.65	2.34	1.43	1.81	2.54	1.56	1.96	2.74
∞	1.28	1.64	2.33	1.28	1.64	2.33	1.28	1.64	2.33

Note that for the case of $n = \infty$, the confidence is one-hundred percent, since one has the entire data population and not just a sample subset. One can therefore calculate with 100 percent confidence the population's true mean and standard deviation. For this special case, the K normal tolerance factors become the z_α percentage points of the standardized normal distribution. Then

$$\text{NTL}_x(\infty, \beta) = \mu_x + (z_\alpha * \sigma_x) \quad (5)$$

where $\alpha = 1 - \beta$, μ_x is the true mean, and σ_x is the true standard deviation of x .

Lognormal Distributions and the NTL

As stated, the Normal Tolerance Limit method should be applied to normally distributed random variables. There is much evidence that many data sets applicable to spaceflight vibroacoustic data are not normal but indeed lognormal (refs. 5 to 11). Therefore, one may still use the NTL method on these data by applying a logarithmic transformation, as follows

$$y = \log_{10} x \quad (6)$$

$$\text{NTL}_y(n, \beta, \gamma) = \bar{y} + (K_{n, \beta, \gamma} * s_y) \quad (7)$$

where \bar{y} and s_y are the sample mean and standard deviation of y . The normal tolerance limits in the original units of x may then be recovered from

$$\text{NTL}_x(n, \beta, \gamma) = 10^{\text{NTL}_y(n, \beta, \gamma)} \quad (8)$$

The transformation is useful for vibration spectra in g^2/Hz and shock response spectra in g when the random source of interest is thought to be lognormally distributed.

Acoustic SPL data in dB can generally be used to obtain NTL directly, without transformation. The reason is that pressure sources are assumed lognormal, implying that SPL dB data are normally distributed due to their calculation via

$$\text{SPL(dB)} = 10 \log_{10} \left(\frac{p}{p_{\text{ref}}} \right)^2 = 20 \log_{10} \left(\frac{p}{p_{\text{ref}}} \right) \quad (9)$$

Typical NTL levels used within the aerospace industry (ref. 12) are:

1. Maximum Expected Environment: NTL ($\beta = 0.95, \gamma = 0.50$) level = P95/C50; MEE levels used as basis for acceptance level testing.
2. Extreme Expected Environment: NTL ($\beta = 0.99, \gamma = 0.90$) level = P99/C90; EEE levels used for qualification level testing.

The Bootstrap Method

Background

Advances in computational speed and cost in the 1970s permitted numerous advances in statistical theories and methods. The Bootstrap, developed by Efron, is one of these methods. In reference 13, Efron defines the problem: “given a random sample $X = (X_1, X_2, \dots, X_n)$ from an unknown probability distribution F , estimate the sampling distribution of some prespecified random variable $R(X, F)$, on the basis of the observed data X .” The Bootstrap allows one to assess the accuracy and uncertainty of estimated parameters from small samples, without any prior assumptions about the underlying distribution. The data need not be normally distributed (as in the NTL method). The Bootstrap also has the advantage of allowing assessment of parameters that may not be mathematically expressible in simple terms, for example, the median of a set of data.

The method consists of repeatedly forming Bootstrap samples (perhaps as few as 100 for parameter estimation and up to a few thousand for confidence interval estimation) of the same size as the original data sample. The elements of each Bootstrap sample are randomly chosen from the original data, with replacements. Thus, a particular sample data point may be chosen several times or perhaps not at all in any particular bootstrap sample. The parameter of interest is then evaluated from each of the bootstrap samples generated. The numerous bootstrap replicates of the parameter can be used to estimate a probability distribution for the parameter. This is an estimate of the parameter sampling distribution, and from this distribution, confidence intervals may be approximated. An overview of the Bootstrap method is given in reference 14.

Bootstrap Method

The following steps define the basic Bootstrap process.

1. Start with data, $X = (x_1, x_2, \dots, x_n)$, a sample set taken from the population with unknown probability distribution F .
2. Select a bootstrap sample of the data by randomly sampling, with replacements, the data X . The bootstrap replicate, X_b , should be of size n . For example, if $X = (x_1, x_2, x_3, x_4, x_5)$, $n = 5$, then one bootstrap replicate might be $X_b = (x_3, x_5, x_3, x_4, x_1)$.
3. Compute the measure of interest θ ; for example if $\theta = \text{mean}(X)$, then $\theta_b = \text{mean}(X_b) = \bar{X}_b = \frac{1}{n} \sum_{j=1}^n X_{bj} = \frac{(x_3 + x_5 + x_3 + x_4 + x_1)}{5}$
4. Repeat steps 2 and 3 numerous times to obtain a large number (B) of bootstrap samples and use each to compute bootstrap replicates of the measure of interest $\theta_B = \{\theta_{b_1}, \theta_{b_2}, \theta_{b_3}, \dots, \theta_{b_B}\}$.
5. Form an empirical cumulative distribution function (CDF) of the measure of interest.
6. Confidence limits or intervals for the measure of interest may then be based on the CDF. For example, the $(1-\alpha) \times 100$ percent level of confidence is found at the $(1-\alpha) \times 100$ percent percentage point of the CDF.

The Titan IV Liftoff Acoustic Database

Objective

For a given set of Titan IV acoustic data, the P95/C50 and P99/C90 specifications are derived using the traditional Normal Tolerance Limit method. The goal of this paper is to apply the Bootstrap method to the same data to compute the “equivalent” bootstrap derived levels. Finally the results from these two methods will be compared.

Acoustic Database Description

Seventeen acoustic microphone measurements from liftoff events from six different launches of the Titan IV expendable launch vehicle were used for this analysis. This dataset, corresponding to flight measurements made inside the Titan IV payload fairing (PLF) at the spacecraft’s location, was studied extensively by NASA in the mid-1990s (ref. 15 to 17) to support the launch of the Cassini spacecraft to Saturn in 1997. Figure 1 illustrates the Cassini spacecraft within the Titan IV launch vehicle’s PLF. Understanding the acoustic environment at PLF zones 7 through 10 was of critical interest.

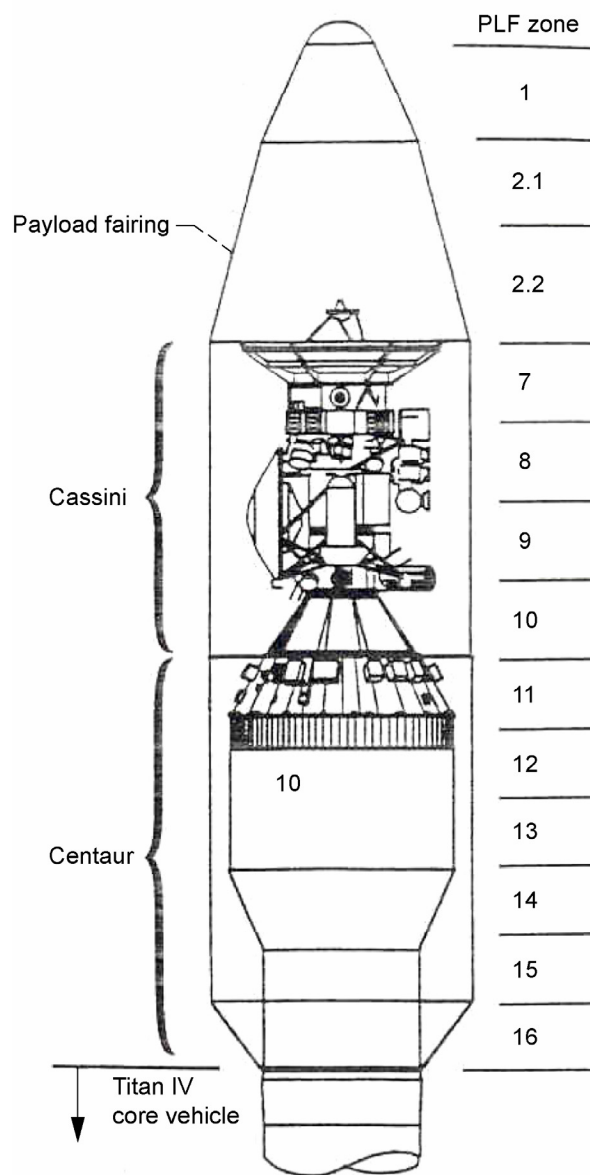


Figure 1.—(left) The Cassini spacecraft configured inside the Titan IV payload fairing; (top right) Titan IV payload fairing in acoustic test chamber; (lower right) Cassini spacecraft mockup used in acoustic test chamber.

Figure 2 shows all 17 SPL measurements plotted together. It is from this collection of data that test levels will be derived to set MEE and EEE levels. The mean, using a straight numerical average of the dB levels at each frequency, is seen as the lowest line in figure 2.

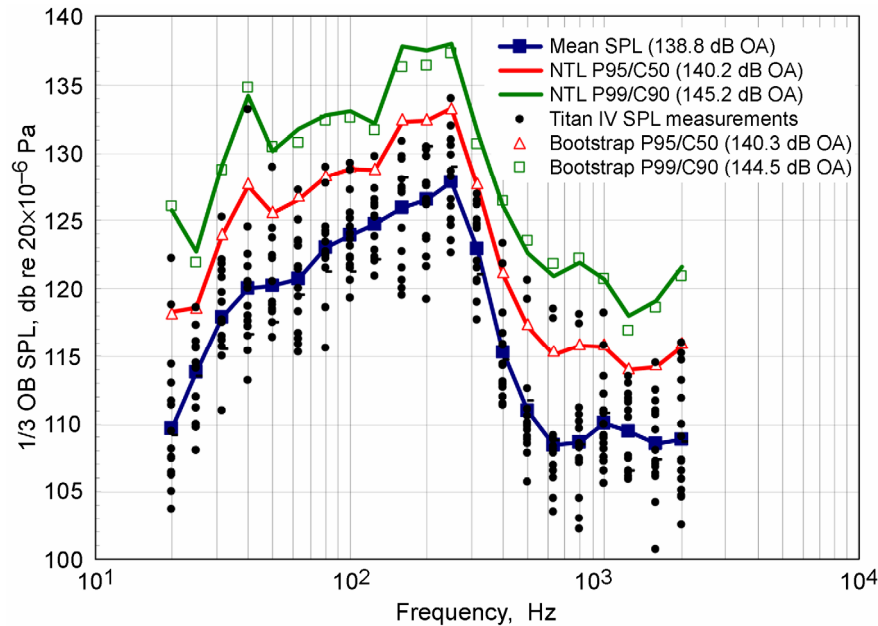


Figure 2.—Titan IV acoustic flight database with comparison of normal tolerance limit and bootstrap statistical levels.

Application of the Normal Tolerance Limit Method

Applying the NTL to this dataset is straightforward. The number of data samples, n , is 17. Referring to table 1 the K factors for NTL ($n = 17$, $\beta = 0.95$, $\gamma = 0.50$) is 1.68, and for NTL ($n = 17$, $\beta = 0.99$, $\gamma = 0.90$) is 3.14. Knowing the K factor, along with the mean and standard deviation, application of the NTL definition (eq. (3)) yields the results shown as the middle (NTL P95/C50) and upper (NTL P99/C90) lines in figure 2.

The assumption in using the NTL method is that the data come from a normal distribution. In this case, it is assumed that the SPL data (in dB) are normally distributed. If this is not the case, the resulting levels may be in error.

The P95/C50 level represents the MEE or maximum expected environment. On average (C50) one would expect 95 percent (P95) of the data to be at or below this P95/C50 level. There are 23 data points that actually exceed the P95/C50 level. Since there are 357 data points in total (17 microphones \times 21 OTOB frequencies) that means that for this particular data set $(357 - 23)/357$ or 93.6 percent of the data actually lie at or below this P95/C50 level. This is indicative that the NTL P95/C50 level is performing as expected. The slight difference from the theoretical 95 percent may be due to the assumption that the SPL dB data is normally distributed, or it may simply be due to the randomness of the events themselves.

For this particular data set, the P99/C90 level should encompass all (or almost all) the flight data if it behaves as expected. Per reference 12 for P99/C90 “there is 1 chance in 10 of exceeding the level once in 100 flights.” Since for this data set we have 357 “flights” the odds are that no flight data should exceed the calculated NTL P99/C90 level. Figure 2 indeed bears this out.

This case points out an advantage of using the NTL P99/C90 level as a qualification ground test level. The design of the spaceflight hardware will be tested out (i.e., qualified) to levels that it should rarely see in flight. Note however that there is one data point at 40 Hz whose level does approach the NTL P99/C90 level. The disadvantage of course is the higher test levels. In this particular case, the NTL P99/C90 levels range from 7.4 to 16.1 dB above the mean of the data depending on the OTOB frequency, with an average increase of 11 dB.

Application of the Bootstrap Method

The Bootstrap method was implemented through the use of MATLAB (The MathWorks, Inc.) coding. It was written to enable the code to generate the statistics of interest for this paper, namely the bootstrap “equivalent” to the NTL P95/C50 and P99/C90 levels. In lieu of a NTL K factor, the bootstrap equivalent P95/C50 and P99/C90 levels were computed by using the bootstrap replicates of the statistics of interest themselves.

The statistics of interest for this bootstrap analysis are the P95 and P99 probability levels. This requires that the bootstrap mean and bootstrap standard deviation be jointly used as a bootstrap pair to compute the desired probability (P) level. In the MATLAB code at each OTOB frequency, a bootstrap replicate set is made that consists of 17 data points randomly selected (with replacement) from the original 17 Titan IV acoustic SPL data points. Next the mean and standard deviation of this bootstrap replicate set is computed. This process of creating bootstrap replicates and generating the bootstrap values for the statistics of interest is repeated numerous (nr) times. Figure 3 shows an example of $nr = 1000$ bootstrap replicates (for the 1000 Hz OTOB frequency data). This figure illustrates the distribution of the replicate pairs of mean and standard deviation values.

The distribution of the original Titan IV flight data itself was used to describe the CDF of the data. To do this the original set of the 17 flight data measurements were converted to standard normal format for each OTOB frequency. This z -value was calculated by taking each data point and first subtracting out the sample mean, and secondly by dividing by the sample standard deviation.

The z -values were calculated for all 17 microphones at all 21 OTOB frequencies. Ideally an individual CDF would be formed from the z -values corresponding to each OTOB frequency. However, due to the limited number (17) of original measurements, this would not provide the necessary resolution

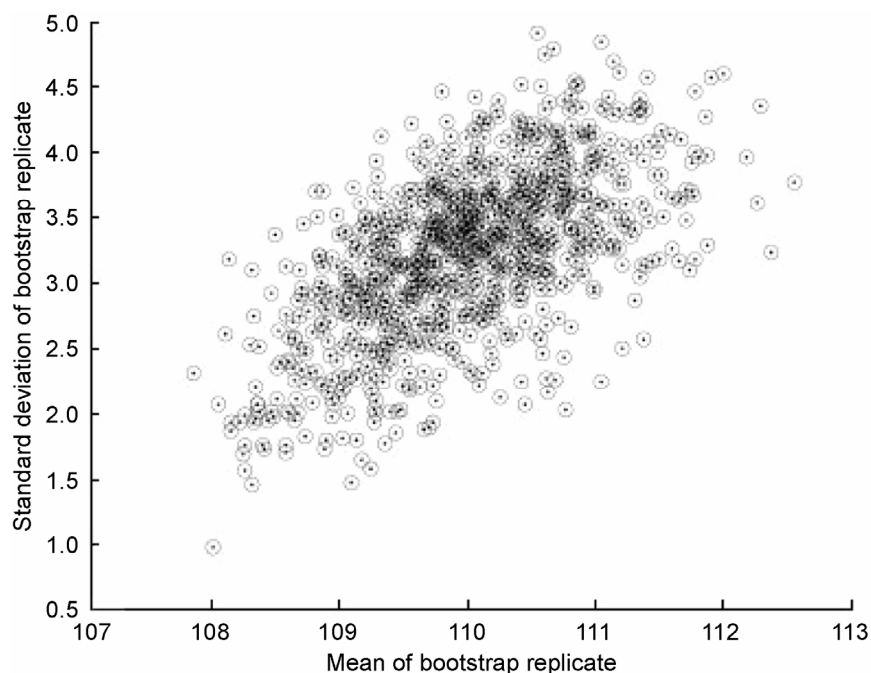


Figure 3.—Illustration of the bootstrap method's distribution of replicate pairs of mean and standard deviation. Frequency = 1000 Hz; Number of bootstrap replicates = 1000.

required to calculate high probability levels from the CDF. Therefore it was decided to combine the z -values at all the OTOB frequencies to form the best estimate of the CDF for this entire data set. Doing this created 357 z -values (17×21) which does provide enough points in the CDF to resolve the P95 and P99 points. Similar approaches of combining data from multiple frequencies have been successfully done for previous NASA programs (refs. 17 and 18).

Combining the z -values at all OTOB frequencies assumes that the data's distribution is the same for all OTOB frequencies. This may be unlikely, but the introduction of this possible error is noted, but accepted as necessary in order to proceed. Some recent work indicates that this assumption may be reasonable (ref. 19). The cumulative distribution function for the Titan IV acoustic SPL database was formed from the z -values. The resulting CDF of the Titan IV data is reasonably similar to that for a normal distribution. Table 2 shows a comparison of these CDFs at the probability levels of interest.

TABLE 2.—PROBABILITY VALUES DERIVED FROM CDF

Level	Titan IV database	Theoretical normal
P50 (mean)	-0.9826	0
P95	1.786	1.645
P99	2.481	2.33

The $P95 = 1.786$ and $P99 = 2.481$ are the z -values used in the MATLAB code for this Titan IV dataset. For each bootstrap replicate the following levels are computed,

$$P95 = \text{bootstrap replicate mean} + (1.786 \times \text{bootstrap replicate standard deviation})$$

$$P99 = \text{bootstrap replicate mean} + (2.481 \times \text{bootstrap replicate standard deviation})$$

At this point, the MATLAB code has calculated nr values of the P95 and P99 levels at each OTOB frequency. To compute the confidence limit these values are sorted, then the percentage point of the distribution corresponding to the desired confidence level is selected.

Figure 2 shows the Bootstrap P95/C50 result (open triangles) for $nr = 1000$. The comparison of this with the NTL P95/C50 level shows no significant difference. This Bootstrap P95/C50 level would represent the MEE or maximum expected environment. As before, one would expect on average (C50) that 95 percent (P95) of the data would be at or below this level. There are 20 points that exceed the bootstrap P95/C50 level. This leads to $(357-20)/357 = 94.4$ percent of the data being at or below this level. This compares favorably to the 93.6 percent result from the NTL P95/C50 level as described previously. A closer inspection of these data shows that the difference between the 23 exceedences of the NTL level and the 20 exceedences of the bootstrap level is due to insignificant changes on the order of 0.2 dB.

Figure 2 also shows the comparison of the Bootstrap P99/C90 result (open squares) for $nr = 1000$ with the NTL P99/C90 level. There is some visible difference, particularly at 160 to 250 Hz, and at 500 to 630 Hz, but even at these frequencies the levels are within 1.5 dB, and within the test tolerance of most reverberant acoustic test chambers. The NTL P99/C90 result tends to be slightly higher, at most but not all frequencies. Both P99/C90 levels easily envelope the maximum from the 17 flight data measurements, as expected for EEE levels.

It appears that the Bootstrap method provides results similar to those obtained with the NTL method for this particular data set. The advantage of using the Bootstrap method is that it makes no assumptions on the distribution of the underlying data. The disadvantage is the additional computational effort required to perform the analysis. The MATLAB runtimes are very short, on the order of seconds and minutes, however there is setup time involved which is long compared to the "hand-like" calculation time of the NTL method.

Summary and Conclusions

The focal comparison of this paper examines acoustic test specification via the NTL method versus test specification via the Bootstrap. For the data considered here, the two methods produce extremely close results as illustrated in figure 2.

NASA and the aerospace industry have traditionally used the NTL to define the MEE and the EEE levels from available flight data. The MEE is used for acceptance testing of flight hardware, and is defined to be P95/C50. The EEE is used for qualification testing of hardware designs, and is defined as P99/C90. (Note that an alternative method used to set qualification test levels is the statistically derived MEE level plus some specified margin, e.g., +3 or +6 dB). By definition the NTL assumes that the data are normally distributed. This assumption has been verified with some previous aerospace acoustic SPL data and is generally considered safe to assume for acoustic SPL data in dB. If the assumption is not correct, as may be the case for random vibration response data, transformation techniques should be employed to accurately use the NTL method.

The Bootstrap Method is a subsampling statistical procedure that uses replicates of the original data to allow estimates of parameters and confidence intervals to be made. The Bootstrap makes no prior assumption regarding the distribution of the data.

Both these methods were applied to the Titan IV acoustic SPL liftoff database consisting of 17 flight acoustic microphone measurements. As shown in this paper, the resulting NTL and Bootstrap P95/C50 and P99/C90 levels were remarkably close. This is believed to be due at least in part to the apparent normality of the sample flight data utilized. Both methods resulted in (test) levels that would perform as MEE and EEE levels should.

The fact that these results are so close for these two methods provides added confidence in the NTL method approach and results that NASA has traditionally used in the past and continues to use today.

Based on the analysis performed for this paper on this particular flight data set, it appears that both the NTL and Bootstrap methods are valid methods to predict the MEE and EEE levels for acoustic SPL data in dB. The NTL method is well established within the aerospace industry and is quick to calculate. However, unlike the NTL method the Bootstrap method does not assume that the sample data come from a normal distribution.

Future investigations comparing test specification obtained via these two methods on other data sets may prove informative. Of particular interest would be applying this to a known non-normal data set to see how the bootstrap results would compare with the NTL results. Examples of this data type might be random vibration response data, shock response spectrum data, or acoustic pressure (in Pascal units, not in dB). If such a comparison were performed, it might, however, be necessary to employ some advanced methods of the bootstrap such as Efron's BCa method (ref. 14), which adjusts the confidence interval limits based on two factors called the bias-correction and the acceleration.

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